RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2018
SECOND YEAR [BATCH 2017-20]
MATHEMATICS FOR INDUSTRIAL CHEMISTRY [General]
Paper : III

Date : 22/12/2018
Time : 11 am – 2 pm
Full Marks : 75

[Use a separate Answer Book for each group]

Group – A

Unit-I

Answer any five questions from Question nos. 1 to 8 : [5×5]

1. Show that the area of the triangle formed by the straight lines $ax^2+2hxy+by^2=0$ and $lx+my=1$ is $\dfrac{\sqrt{h^2-ab}}{am^2-2hlm+bl^2}$. (5)

2. Reduce the equation $x^2-2xy+y^2+6x-14y+29=0$ to its canonical form. (5)

3. Find the equation of the pair of tangents from the point (–2, 1) to the parabola $y^2=8x$, and also find the angle between them. (3+2)

4. If the straight line $\mathrm{r}\cos(\theta-\alpha)=\mathrm{p}$ touches the parabola $\dfrac{l}{r}=1+\cos\theta$, show that $p=\dfrac{l}{2}\sec\alpha$. (5)

5. Find the equation of the plane which passes through the point (–1, 2, 0) and is perpendicular to each of the planes $x+2z-4=0$ and $3x-2y+5z-6=0$. (5)

6. Determine the value of m so that the lines $\dfrac{x-1}{2}=\dfrac{y-4}{1}=\dfrac{z-5}{2}$ and $\dfrac{x-2}{-1}=\dfrac{y-8}{m}=\dfrac{z-11}{4}$ may intersect. (5)

7. a) Find the angle between the two lines given by $6x^2-5xy-6y^2=0$. (3)
   b) Find the angle between the planes $x+2y-z=0$ and $2x+3y-z-4=0$. (2)

8. A sphere of constant radius r passes through the origin and meets the axes in A, B, C. Prove that the centroid of the triangle ABC lies on the sphere $9\left(x^2+y^2+z^2\right)=4r^2$. (5)

Unit-II

Answer any five questions from Question nos. 9 to 16 : [5×5]

9. Prove that $\mathbb{R}^2$ forms a vector space over $\mathbb{R}$ under the operations defined by
   $(a,b)+(c,d)=(a+c,b+d)$ and
   $x(a,b)=(ax,bx)$ for any $(a,b),(c,d)\in\mathbb{R}^2$ and $x\in\mathbb{R}$. (5)

10. Determine whether $S=\left\{(x_1,x_2,x_3)\in\mathbb{R}^3 : x_1x_2=0\right\}$ forms a subspace of $\mathbb{R}^3$. Justify your answer. (5)

11. Let V be a vector space over the field F and α, β, γ are three linearly independent vectors in V. Then show that the set $S = \{\alpha, \alpha+\beta, \alpha+\beta+\gamma\}$ is also linearly independent in V. (5)

12. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T(x_1, x_2, x_3) = (x_1 - x_2, 2x_3, x_2 + 2x_3)$. Find $[T]_\beta$, where $\beta = \{(1,0,0), (0,1,0), (0,0,1)\}$. (5)

13. Consider the map $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3, x_1 + x_2, 0)$.
    - i) Show that T is a linear transformation. (3)
    - ii) Is T one-one? Justify. (2)

14. Let V be the vector space of all 2×2 real matrices over $\mathbb{R}$. Show that $\dim(V) = 4$. (5)

15. Find a basis of $\mathbb{R}^3$ containing (1, 1, 2) and (3, 5, 2). (5)

16. Show that $\{(1,-3,2), (2,4,1), (1,1,1)\}$ forms a basis for $\mathbb{R}^3$. (5)

Group – B

Answer any five questions from Question Nos. 17 to 24 : [5×5]

17. Obtain the differential equation of the system of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ in which λ is the arbitrary parameter and a, b are given constants. (5)

18. Solve: $x \cos\frac{y}{x}\,(y\,dx + x\,dy) = y \sin\frac{y}{x}\,(x\,dy - y\,dx)$. (5)

19. Consider the following differential equation: $(y^2 + 2xy)dx - x^2 dy = 0$.
    - a) Show that the above differential equation is not exact. (2)
    - b) Find the value of n, such that y^n becomes an integrating factor of the given differential equation. (3)

20. Solve: $\frac{dy}{dx} = \frac{y-x+1}{y+x+5}$. (5)

21. Solve: $(xy \sin xy + \cos xy)\,y\,dx + (xy \sin xy - \cos xy)\,x\,dy = 0$. (5)

22. Consider the differential equation $\frac{dy}{dx} + P(x)y = Q(x)y^n$.
    Show that it can be reduced to a linear equation by the substitution $z = y^{1-n}$. (5)

23. Find the orthogonal trajectory of family of parabolas $y^2 = 4a(x+a)$, a being the parameter of the family. (5)

24. Find the general and singular solution of the differential equation $y = px + ap(1-p)$, where $p = \frac{dy}{dx}$. (5)

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